

Shape control of swept surface with profiles

Guo-ping Wang^{a,*}, Jia-guang Sun^b

^aDepartment of Computer Science and Technology, Peking University, Beijing 100871, People's Republic of China

^bDepartment of Computer Science and Technology, Tsinghua University, Beijing 100084, People's Republic of China

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Abstract

Shape control techniques for swept surface using profiles are developed in this paper. The characteristics of the swept surface are used to reduce the shape control of the swept surface to modify the contour shapes by using profiles. This method is more convenient and more intuitive for the user. The deformed region of the contour is defined by deformation rules proposed in this paper. The robustness and efficiency of this technique are verified by many examples implemented in the commercial geometric modeling software GEMS 5.0 developed by the CAD Center of Tsinghua University. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The sweeping operator is a powerful function in geometric modeling for describing the shapes of complex surfaces or solids. The swept surface or volume can then be described by orbit sets which are formed by curves, surfaces or objects moving through a spine (also called as a space trajectory). The method is simple and efficient, and requires only the specification of the moving object and of the spine along which the object moves. The swept surface can be expressed as follows:

$$S(u, v) = r(v) + C_1(u, v)\mathbf{B} + C_2(u, v)\mathbf{N} \quad (1)$$

Where $r(u)$ is the spine or trajectory, $c_1(u, v)$ and $c_2(u, v)$ are the planar contours which can be deformed and twisted along the spine, and \mathbf{N} and \mathbf{B} are the unit vectors of a moving frame along the spine. The method has many good characteristics such as:

- Reduces the surface design to curve modification so that the design problem is simplified.
- Reduces the depth of the CSG feature tree in the geometric modeling system and reduces the operator steps in the product design so that the design efficiency is improved.
- The complex shape can be described intuitively in terms of a contour moving along a spine with twist and deformation.

Many commercial geometric modeling systems contain this powerful function [9], and many papers have been written on this topic [1–7]. But, although this operator is useful in commercial geometrical modeling systems, the shape of the swept surface must often be modified interactively and a good modification method can reduce the interactive time and improve the design efficiency during production design. Therefore, a flexible and convenient method is needed for the user to deform the shape of the swept surface.

The non-rational expression in Eq. (1) for the swept surface is usually approximated by a NURBS surface for compatibility with the data structures and algorithms in the geometric modeling systems. With a NURBS surface, the shape control technique, such as control point relocation and weight modification can be adopted. But a production designer or a user of the commercial CAD software may be unfamiliar with the mathematical basis of NURBS surfaces, so these surface modification techniques may be too professional, less intuitive, and difficult to master. Other methods can be used to control the shape of the swept surface using a profile or a transform matrix [1,3,6,9]. But the shape deformation is limited to the scaling deformation of the contour on the local moving frame along the spine. Therefore, the ability to modify the swept surface shape is limited so cannot utilize all of the advantages of the sweep method and cannot provide an intuitive and convenient method for shape modification.

This paper presents some shape control methods. But many problems need to be resolved such as how to define the deformed point on the surface and how much of the

* Corresponding author.

E-mail address: wgp@graphics.pku.edu.cn (G. Wang).

region on the contour is deformed by the profiles (referred to as the deforming curve). These issues will be discussed in Sections 2 and 3. Furthermore, some deformation rules introduced in Section 4 are used to determine the deformed regions on the contours according to the shape of each profile curve. Finally, several examples are given that have verified the robustness and efficiency of the method implemented in the commercial geometric modeling software GEMS 5.0 which was developed by the CAD Center of Tsinghua University.

2. Terms and definitions

The contour $C(u, v)$ in Eq. (1) can be considered to be the contour $C(u)$ deformed or twisted by the profile curve $\tilde{f}(t)$ or by a twist angle along the spine. The profile $\tilde{f}(t)$ can be reparameterized with the spine $r(v)$ such that $\tilde{f}(t) = \tilde{f}(r(v))$, which is simply denoted as profile curve $f(v)$. One effective method for constructing the swept surface [2,6,7] is to form the local moving frame at the selecting points on the spine and to set the intermediate contours on these local moving frames by coordinate transformation. The swept surface is then constructed by skinning all intermediate contours. Therefore, the shape of the swept surface can be reduced to the determination of the shape of the intermediate contours.

Construction of the swept surface involves several issues:

1. Because the distribution of the intermediate contours affects the shape of the swept surface in the skinning procedure, the intermediate contours must be distributed along the spine so that the contour distribution can reflect the shape change of the spine and the profiles.
2. The profiles effect on the swept surface can be reduced to just the effect of the contours (Fig. 1). But the maximum affecting points (referred to as the *deforming points*) on the profiles and the maximum affected points (referred to as the *deformed points*) on each intermediate contour must be properly defined relative to the distribution of the intermediate contours.
3. The deformed distances between each intermediate contour and the profiles must be determined because the amount of deformation of each contour is proportional to the distances.
4. The deformed region on each intermediate contour (or even on the swept surface), which can be relocated by each profile, must be conveniently and intuitively specified by the user.

For the first issue, the subdivided algorithm of the curve [5,7] can be generalized to simultaneously subdivide the spine and profiles to obtain the split points on the spine and the profiles. In the recursive-subdivision algorithm, the distributing of the split points reflects the shape of the spine and the profiles. If the shape of some region on the spine or on

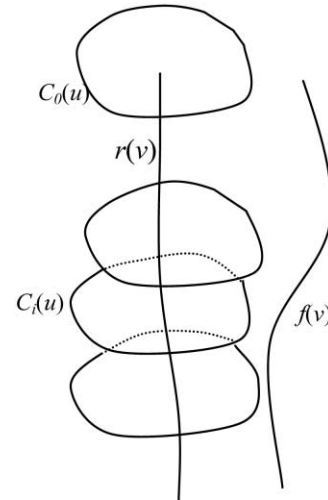


Fig. 1. Profile effect on the distribution of contours.

the profile changes rapidly, the split points on this region must be densely distributed. Otherwise, the split points can be sparsely distributed. The intermediate contour and its moving frame are then located at the split point on the spine.

For the second and third issues, the users desires can be met using two proposed methods with specifically defined terms, such as deformed points, deforming points, deforming direction and deformed distance. The first method is called the parameter method, and the second is called the intersection method. These two methods can generate different deformations on the swept surface.

2.1. Parameter method

For the curve subdivision algorithm, the terms can be defined as:

Initial contour plane: the plane including the initial contour;

Deforming point: the split point on the profile;

Deformed point on the initial contour is closest to the projected point of the initial point of the profile on the initial contour plane (Fig. 2).

The deformed points on the other intermediate contours have the same parameters as the first deformed point in the initial contour. Since split points on the spine and on each profile form one point pair, each deforming point and the corresponding deformed point on each profile can form a point pair for each profile

Deforming distance of ONE split point on the profile — the distance between the initial projected point of the profile and the projected point of ONE split point of the profile on the initial contour plane. The length of the projection on the initial plane of the vector from the initial point of the profile to ONE split point of the profile is the deforming distance of ONE split point of the profile.

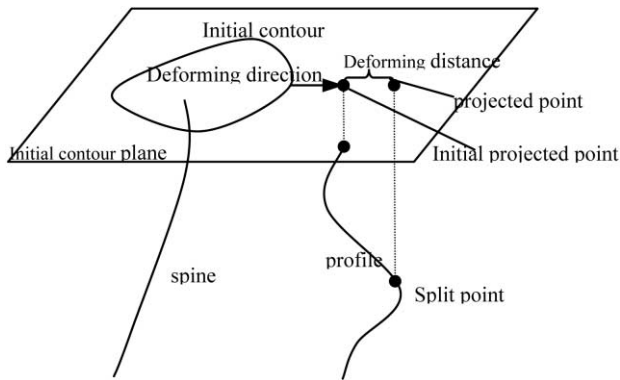


Fig. 2. Definitions of the terms for the parameter method.

Deforming direction — the vector from the deformed point of the initial contour to the initial projected point of the profile on the initial contour plane.

The deforming direction is fixed to the local moving frame together with the intermediate contour, so the other deformed points on all intermediate contours and their deforming directions can be defined.

Notice that the deforming distance is dependent on the projected increment of the split points of the profile on the initial contour plane as shown in Fig. 2, and may be negative in accordance with the profile shape change. Some examples of this method are shown in Figs. 13, 16, 18 and 21.

For multi-profiles, the number of deformed points on each intermediate contour is the number of profiles.

2.2. Intersection method

The terms for the intersection method are defined as:

Deforming point and deformed point: if the plane including the intermediate contour L intersects with one profile, then the intersection point on this profile is called the *deforming point* of the profile and the point on L that is closest to the deforming point on this profile is called the *deformed point* of L . The deforming point and the deformed point form a point pair for each profile.

Deforming distance: the distance between the deformed point and the corresponding deforming point.

Deforming direction: the vector from the deformed point to the deforming point (Fig. 3).

Intermediate contours which do not intersect with the profile are not deformed by the profile. The intersection method is especially recommended for 2D spines or spines which change shape slowly, e.g. Figs. 7–12 and 17.

These two methods have some important differences. The parameter method determines the deforming distance from the projected increment of the splits of the profile on the initial contour plane and considers the relative shape change of the profile. The parameter method then determines the deforming direction and the deformed point on the contours

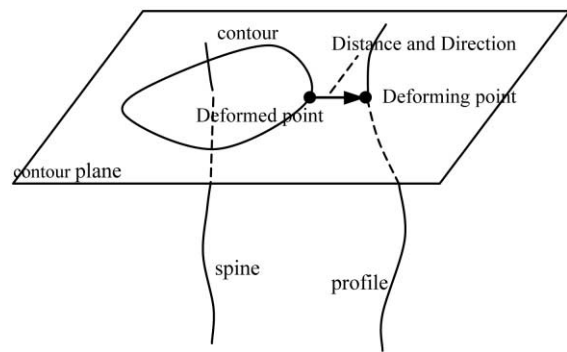


Fig. 3. Definitions of the terms for the intersection method.

from the initial contour and the projection of the initial point of the profile. The deforming direction is fixed in the local moving frame along the spine. The intersection method determines all terms, such as the deforming point, deformed distances and so on, from the intersection procedure between each intermediate contour plane and the profiles.

3. Local deformation of contours

From the unified NURBS expression for the swept surface, the shape of the swept surface can be changed by the relocation of its control points or by modifying its weights, but the procedure is not intuitive for users. The characteristics of the swept surface can be used to reduce the local deformation of the surface to deform the intermediate contours. The local region on the swept surface which is to be deformed is defined by the user. The following defines two types of deformation regions on the contours which are needed by the users.

3.1. Influence region of the local deformation

From the local properties of an NURBS curve, removing a point on the curve will affect the region near this point on the curve (Fig. 4). The deformed point on the contour, together with its deforming direction and its deforming distance can be computed from Section 2, then the affected region for this deformed point on the contour can be repositioned by

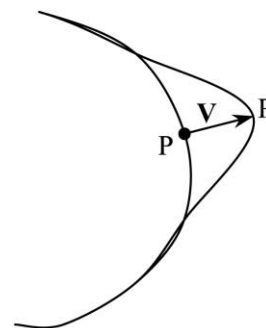


Fig. 4. Local deformation curve.

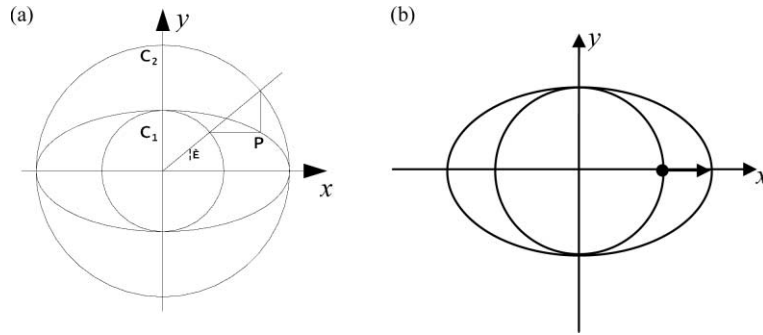


Fig. 5. (a) Generation of an ellipse. (b) The ellipse as the e-offset of a circle.

removing the closest control point of the deformed point on the contour [5]. To maintain the smoothness of the deformed contour, two control points should be removed near the deformed point on the contour. Suppose we want to remove the deformed point $P = C(\bar{u})$ along the deforming direction \mathbf{V} with the deforming distance d , then the region near P on the contour can be relocated by translating both P_i and P_{i+1} along direction \mathbf{V} . Here, \bar{u} is the parameter of the deformed point on the contour (assuming $\bar{u} \in [u_i, u_{i+1})$). For $0 \leq \gamma \leq 1$, then

$$\hat{P}_i = P_i + (1 - \gamma)\alpha\mathbf{V}, \quad \hat{P}_{i+1} = P_{i+1} + \gamma\alpha\mathbf{V} \quad (2)$$

The parameter γ is allowed to vary, but the smoothness of the curve is maintained by setting $\gamma = (\bar{u} - t_i)/(t_{i+1} - t_i)$, where $t_i = (1/k) \sum_{j=1}^k u_{i+j}$, $i = 0, \dots, n$, is the i th node of the knot vector, and $\{u_i\}_{i=0}^{n+k}$ is the knot vector of the k th-order NURBS curve. Obviously, t_i is closest to the parameter of point P_i among the knots vectors. From the translation invariance property of the NURBS curve

$$|\hat{P} - P| = d = |(1 - \gamma)\alpha\mathbf{V}R_{i,k}(\bar{u}) + \gamma\alpha\mathbf{V}R_{i+1,k}(\bar{u})|$$

which implies that

$$\alpha = \frac{d}{\|\mathbf{V}\| |(1 - \gamma)R_{i,k}(\bar{u}) + \gamma R_{i+1,k}(\bar{u})|} \quad (3)$$

Then the new control points for the locally deformed curve are computed from Eq. (2). For more details, see Piegl and Tiller [5].

3.2. The ellipse-like offset curve

A curve deformation that is smoother than the local deformation can be obtained by naturally deforming along a given some direction. From the construction of the ellipse, the ellipse can be considered to be a generalized offset from a circle with a smooth deformation. So one type of generalized offset curve for the contour deformation can be given as follows:

Definition 1. For a regular curve $C(u)$ and a unit offset direction \mathbf{V} , the *ellipse-like offset curve* $C_e(u)$ with the offset distance d is determined by

$$C_e(u) = C(u) + d(\mathbf{n}(u) \cdot \mathbf{V})\mathbf{V} \quad (4)$$

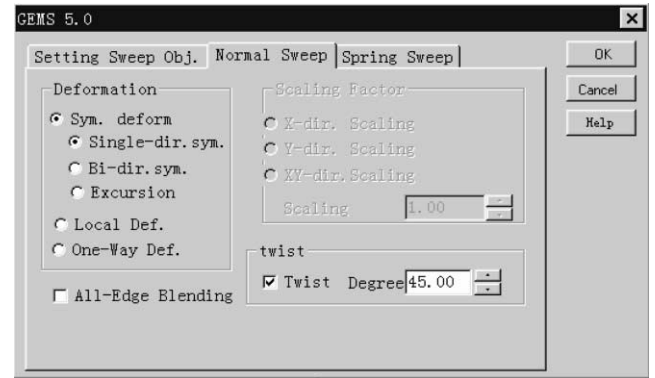


Fig. 6. Sweep operator dialog box.



Fig. 7. Single-direction symmetric deformation rule.

where $\mathbf{n}(u)$ is the unit normal vector of the curve $C(u)$. We denote the ellipse-like offset as e-offset with the usual offset as simply offset. Analogous to that of the usual offset, the offset distance d to determine the outer and inner e-offset can be positive and negative.

As shown in Figs. 5, 7 and 10, the e-offset curve is situated between the origin curve and the usual offset curve. To simplify the expression for the e-offset curve, the parameter u of $C(u)$ is assumed to be the arc-length parameter, so the following properties are straightforward.

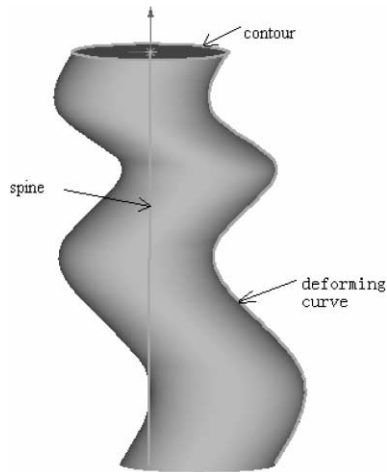


Fig. 8. Excursion deformation rule.

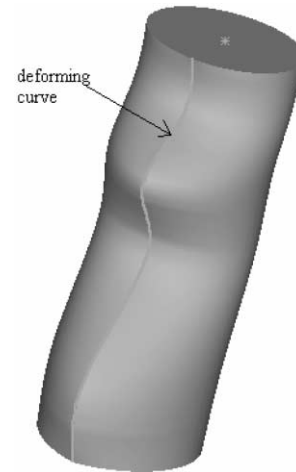


Fig. 9. Local deformation rule.

Property 1. For $C(u)$ and $C_e(u)$, if \mathbf{T} and \mathbf{V} is not parallel to each other, then \mathbf{T} and \mathbf{T}_e have the following relationship:

$$\mathbf{T}_e = (1 + d \cdot \kappa) \mathbf{T} + (d \cdot \kappa) \mathbf{U}$$

where \mathbf{T} is the tangential vector of $C(u)$ and \mathbf{U} is obtained by rotating vector \mathbf{V} through a right angle clockwise along the normal vector of plane π which includes $C(u)$ if \mathbf{T} is directed clockwise along the normal vector of π . Otherwise, \mathbf{U} is obtained by rotating vector \mathbf{V} through a right angle counterclockwise along the normal vector of π .

Property 2. For $C(u)$ and $C_e(u)$, the corresponding tangential vectors \mathbf{T} and \mathbf{T}_e have the following relationship:

$$\mathbf{T}_e \cdot \mathbf{V} = (1 + d \cdot \kappa) \mathbf{T} \cdot \mathbf{V}$$

Therefore, the projection of \mathbf{T}_e and \mathbf{T} on \mathbf{V} is pro rata.

Property 3. For a regular curve, if the normal vector at one point on the curve is parallel to the offset direction \mathbf{V} , this point on the curve has the maximum offset distance. If the normal vector of the curve is perpendicular to the offset direction, then the offset distance at that point on the curve is zero.

Property 4. For a circle with radius r , its ellipse-like offset with the distance d is an ellipse, and the length of the long axis is $2(d + r)$ if $d > 0$.

If the e-offset direction \mathbf{V} is defined as the normal vector $\mathbf{n}(u)$ of the curve, then the e-offset becomes the usual offset. So the e-offset is the generalization of the usual offset. Furthermore, the uniform expression of the data structures and algorithms in the geometric modeling system can be used to develop a NURBS approximation algorithm for e-offset contours.

For the offset of a NURBS contour, one NURBS approximation method is to offset the contour control points

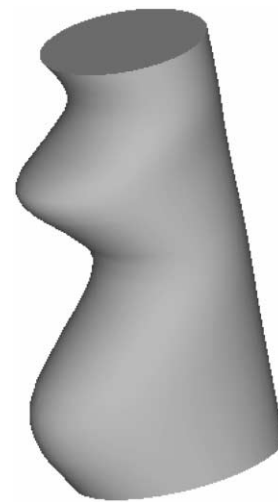


Fig. 10. One-way deformation rule.

(Refs. [3,7], etc.). Here, the offset distance of each control point will be computed to generate a NURBS e-offset contour. Two NURBS approximation algorithms of the e-offset curve will be presented:

First, the number of control points for the e-offset NURBS contour depends on the e-offset distance and the shape of the original curve. So the number of control points differs for each e-offset intermediate contour in the sweeping procedure. If the contour shape is changed sharply, the number of new control points will increase rapidly, so the compatibility of all e-offset intermediate contours needs to be considered for skinning all intermediate contours to construct the swept surface.

Another method is to fix the number of control points for the e-offset step and only consider the key points on the curve, so the offset approximation precision at some positions on the e-offset curve will be reduced some. But since for the concept design in practice the deformed shape of the swept surface does not need to be presented precisely, but

needs to be smooth, the second method is sufficient, then all the e-offset intermediate contours do not need to be compatible, and the design efficiency can be improved.

The second approximation can be defined by the following algorithm. Letting the control points of the k th-order NURBS contour $C(u)$ be $\{P_i\}$, $i = 0, \dots, n$, then point Q_i on the curve most influenced by P_i is near to $C(t_i)$, where t_i is the i th node of the knot vector as discussed in Section 3.1. The e-offset distance d_i and the e-offset vector \mathbf{V}_i can be computed from Section 2, and then the new control point \tilde{P}_i of the e-offset contour can be computed from the following linear equations:

$$C_e(t_i) = \sum_{j=0}^n \tilde{P}_j R_{j,k}(t_i) \quad i = 0, \dots, n \quad (5)$$

where $R_{i,k}(t_i)$ is the rational basis of the k th-order NURBS curve and $C_e(t_i)$ is computed from Eq. (4). The coefficient matrix of the above linear equations is invertible. A unique solution exists and the new control points of the e-offset contour can be computed. The weights and the knot vector of the e-offset NURBS contour are invariable.

Remark. If the contour is a circle or a line, the e-offset contour can be directly computed from the e-offset of the control points. If the contour shape changes rapidly, some knots of the NURBS curve can be inserted to increase the number of contour control points in the preprocessing step to reduce the approximation error. The error estimates for the e-offset approximation and another approximation algorithms are discussed by Wang [8].

4. Deformation rules for the swept surface

For flexible and convenient control of a surface shape by the user, the surface modification region needs to intuitively anticipate by the user. This section discusses the fourth issue described in Section 2, that is, how to deform the contours with profiles and the size of the contour region deformed by the profiles. The different contours regions deformed by the profiles are used to propose five types of deformation rules for the swept surface.

1. *Bi-directional symmetric deformation.* This rule is similar to the offset operator for the 2D curve. The intermediate contour's offset curve is considered to be the deformed contour, and if more profiles occur, the first input profile determines the offset distance described in Section 2. Therefore, ONE profile is sufficient and other profiles will be ignored. See Figs. 11–13.
2. *Single-directional symmetric deformation.* The contour only moves along the deforming direction in either direction. For a smooth deformation, an e-offset operator is used to deform the intermediate contour along the deforming direction, and its deforming distance can be computed as in Section 2 according to the profile shape

for the different intermediate contours. If the deforming directions of different profiles are the same or the reverse direction, only the deforming direction of the first of these profiles will be considered and the others will be ignored. Otherwise, the different deformations along the deforming directions will be superposed on the intermediate contours. See Figs. 17 and 21.

3. *Excursion along a profile.* In this rule, the entire intermediate contour is translated along the deformation direction with the deforming distance according to the profile shape. In this rule, ONE profile is sufficient, and the other profiles in the input profile list will be ignored. See Fig. 8.
4. *Local deformation.* The intermediate contours will be deformed locally at their deformation points along their deformation directions. The locally deformed region on the intermediate contours will be determined by the local deformation method described in Section 3.1. That is, the intermediate contour is deformed as a NURBS curve local deformation. In this rule, all the local deformations along their deforming directions by the profiles will be superposed on the intermediate contour as shown in Figs. 14–16.
5. *One-way deformation.* The deformed region is determined by the e-offset operator, but not total region of the intermediate contour will be deformed. For any point on the intermediate contour, if the angle between the normal vector of the point and the deforming direction is larger than 90° , then the deformation at this point on the intermediate contour will be canceled, so only one side of the contour toward the deformed direction is deformed and the shape of the other side of the contour is unchanged. With this rule, all deformations along their deforming directions by profiles will be superposed on the intermediate contour. See Figs. 10 and 18.

Remark. A Sweep Operator dialog-box in GEMS 5.0 is shown in Fig. 6. With these types of deformation rules, the deformed region, the deformed distance and the deformed type of the swept surface can be selected by the user at will. The deformation on the swept surface along the profiles can be reduced to the deformation of the intermediate contours along the profiles. The contour deformations are controlled along the spine by the profile, which allows non-linear or linear curves. If the user demands a linear deformation of the contour along the spine with a linear profile, then scaling control is more convenient. Therefore, an optional scaling factor is also input in the dialog-box in GEMS 5.0 shown in gray in Fig. 6, which is used in the example shown in Fig. 19.

5. Sweep algorithm with deformation

If twist is considered along the spine, each intermediate

contour is rotated about the tangential vector of the spine with twist angles equally divided according to the spine arc-length ratio. For NURBS curves, the sweep algorithm with deformation and twist can be described as follows:

Step 1. To maintain compatible between the spine and the profiles, simultaneously subdivide the spine and the profiles to obtain the split points on the spine and the profiles.

Step 2. Place the local moving frame at each split point along the spine and set the intermediate contour on this local moving frame. If more initial contours are input by the user, then the contour shapes will change from one initial contour to another initial contour along the spine. The initial contours will be placed at some positions on the spine by the user, or will be placed at equally arc-lengths on the spine as the system default, then the intermediate contours at each split point along the spine can be obtained by linearly interpolating between two adjoint initial contours, as shown in Figs. 12 and 13.

Step 3. Compute the deformed points on each intermediate contour, the deforming points on the profiles, the deforming distance and the deforming direction by the methods described in Section 2. If twist is considered, equally divide the twist angle according to the spine arc-length ratio for each intermediate contour.

Step 4. According to the deformation rule selected by the user, deform each intermediate contour along each profile. If twist is considered, twist each intermediate contour along the spine after performing the deformation procedure.

Step 5. The compatibility of all intermediate contours should be maintained even if the number of control points for the intermediate contour is not the same. Then skin all the intermediate contours to generate the tensor product NURBS swept surface.

Step 6. Error-estimates are made by selecting some isoparametric curves on the skinning surface along the spine to estimate the interpolation-error bounds for some error-estimate sample points on the curves from Eq. (1). If some of the errors are larger than the prescribed error-bound, insert one knot in the spine and return to Step 1; otherwise, error for the Swept surface or solid is acceptable.

Remark. The interpolation-error can be also estimated as developed by Wang [7,8].

6. Examples

The following examples were implemented in the commercial CAD system GEMS 5.0, which was developed by the National CAD Center of Tsinghua University. GEMS 5.0 is a feature-based geometric modeling system with many advanced functions such as drawing with intelligent PDA, solid and surface modeling, assembly, 2D engineering drawing, FEA, sheet-metal and rendering. Many Chinese

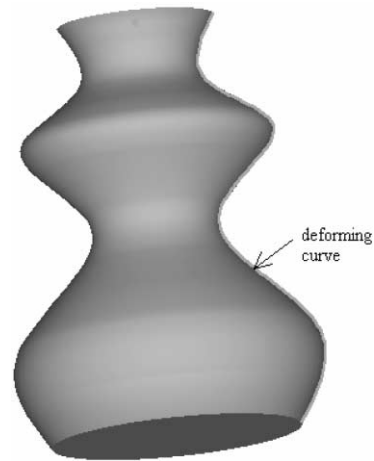


Fig. 11. Bi-directional symmetric deformation rule.

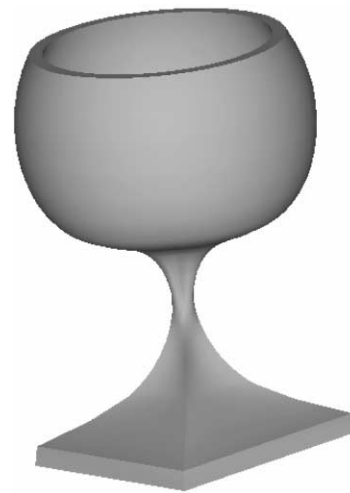


Fig. 12. Cup formed by Bi-directional symmetric deformation rule.

mechanical engineering factories and companies use the system.

Example 1. The contour is a circle, the spine is a line segment, and the profile is a NURBS curve (Figs. 7–11). The shape of the swept solid is deformed by the profile (also called as the deforming curve shown in Figs. 8 and 9).

Example 2. The contours are a circle and a rectangle and the spine is a line segment. The profile is composed of a NURBS curve and a line segment to form a cup (Fig. 12).

Example 3. The contours are a circle and a pentagon and the spine include a line segment, an arc segment and a line segment, which are joined as G^1 continuity. The profile is a NURBS curve. A tobacco pipe is formed (Fig. 13).

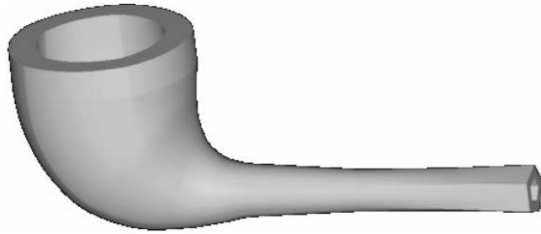


Fig. 13. Pipe formed by the bi-directional symmetric deformation rule.



Fig. 14. Local deformation by two profiles.

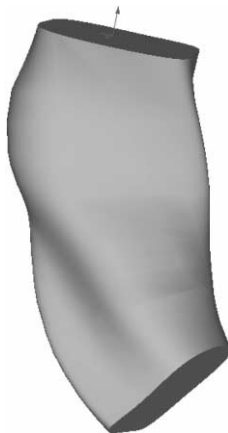


Fig. 15. Twist added to Fig. 14.

Example 4. The contour, spine and profiles are all NURBS curves with the local deformation rule (Fig. 14) and a 90° twist (Fig. 15).

Example 5. The contour is a rectangle and the spine is a fourth-order NURBS curve. One profile is a G^0 curve composed of line segments and other profile is a fourth-order NURBS curve with a twist angle of 45°. The local deformation rule is used (Fig. 16). Notice that if the deforming curve is G^0 continuity, the corresponding deformed surface is also G^0 continuity.

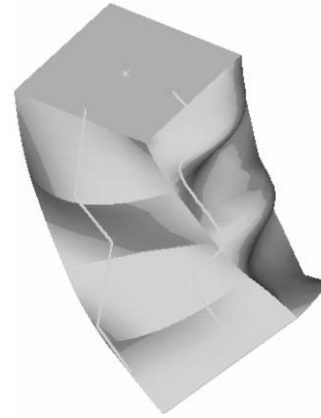


Fig. 16. Local deformation by one C^0 profile and C^1 profile, with twist angle of 45°.

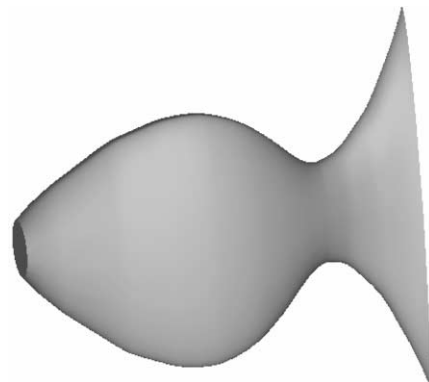


Fig. 17. Fish-like solid formed by two profiles.

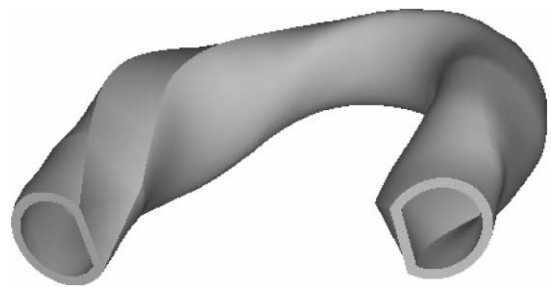


Fig. 18. Twist angle of 360° with one-way deformation rule.

Example 6. The contour is a circle and the spine is a line segment. The two profiles are fourth-order NURBS curves. The single-directional symmetric deformation rule is used to generate a fish-like solid (Fig. 17).

Example 7. The one-way deformation rule with a twist angle of 360°. The spine and the profile are both NURBS curves, and the contours are composed curves (Fig. 18).

Example 8. The contour is an ellipse and the spine is a circle. The linear deformation rule is adopted, and the

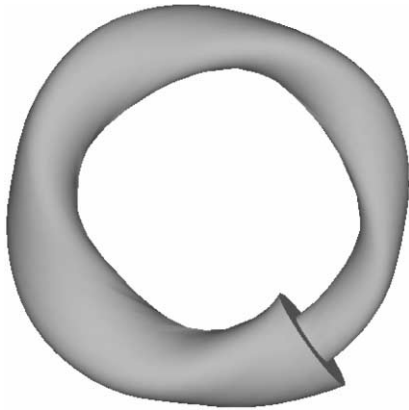


Fig. 19. Mobius-like solid with twist angle of 720° , and scaling of 0.5.

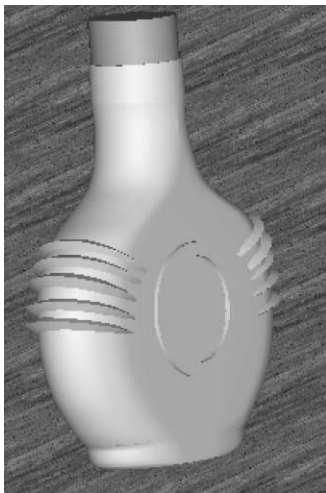


Fig. 20. Bottle generated with single-directional symmetric deformation rule.

scaling is 0.5 with a twist angle of 720° to generate a Mobius-like ring solid (Fig. 19).

Example 9. A bottle (Fig. 20) designed by a user with the Sweep Operator in GEMS 5.0. The profile is a NURBS curve and the contours are a circle and an ellipse with the single-directional symmetric deformation rule.

Example 10. A spoon swept using two NURBS profiles (Fig. 21).

7. Conclusion

Flexible and convenient control of the surface shape by the user in a commercial geometric modeling system is presented using a simple, intuitive deformation method in which the shape control of the swept surface is reduced to



Fig. 21. Spoon swept via two profiles.

modifying the shapes of profiles or the spine. Two user selectable functions, an intersection method and a parameter method, are proposed to define the deforming points on the profiles, the deformed points on each intermediate contour, the deforming distance and the deforming direction, which determine the deformed region and the deformed extent on the swept surface. Furthermore, deformation rules are presented to generate various types of shape deformations on the surface, and an ellipse-like offset operator is presented to deform the contours more smoothly and more naturally. Finally, many examples have verified the efficiency and the robustness of the method in the commercial geometrical modeling system GEMS 5.0 developed by Tsinghua University.

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Guo-ping Wang is an associate professor in the Department of Computer Science and Technology at Peking University. He received his BS and MS degrees from Harbin Institute of Technology in 1987 and 1990, respectively, and PhD degree from the Fudan University in 1997, all in Mathematics. From 1997–1999, he was a postdoctoral researcher in Department of Computer Science and Technology in Tsinghua University at Beijing. His current research interests are in Virtual Reality, Computer Graphics and Computer-Aided Geometric Design.

Jia-guang Sun is a professor in the Department of Computer Science and Technology at Tsinghua University. He is also Director of National CAD Engineering Center at Tsinghua University and Academician of the Chinese Academy of Engineering. He received his BS degree in Computer Science from the Tsinghua University in 1970. From 1982–1986, he was a visiting scholar in UCLA. His current research interests are in Computer-Aided Geometric Design, Computer Graphics and Product Data Management.